Laplace Transform Table

Common Laplace transforms and properties. a, b, c are constants, n is a non-negative integer, u(t) is the unit step and δ(t) the Dirac delta. Region of convergence is given for Re(s).

Basic functions

f(t)	F(s) = $\mathscr{L}\{f(t)\}$	Valid for
1	$1/s$	$\mathrm{Re}(s) > 0$
t	$1/s^2$	$\mathrm{Re}(s) > 0$
t^n $(n = 0, 1, 2, \ldots)$	$n!/s^{n+1}$	$\mathrm{Re}(s) > 0$
t^a $(a > -1)$	$\Gamma(a+1)/s^{a+1}$	$\mathrm{Re}(s) > 0$
$\sqrt{t}$	$\sqrt{\pi}/(2s\sqrt{s})$	$\mathrm{Re}(s) > 0$
$1/\sqrt{t}$	$\sqrt{(\pi/s)}$	$\mathrm{Re}(s) > 0$

Exponentials

f(t)	F(s) = $\mathscr{L}\{f(t)\}$	Valid for
e^{at}	$1/(s-a)$	$\mathrm{Re}(s) > a$
$t \cdot e^{at}$	$1/(s-a)^2$	$\mathrm{Re}(s) > a$
$t^n \cdot e^{at}$	$n!/(s-a)^{n+1}$	$\mathrm{Re}(s) > a$

Trigonometric and hyperbolic

f(t)	F(s) = $\mathscr{L}\{f(t)\}$	Valid for
$\sin(bt)$	$b/(s^2+b^2)$	$\mathrm{Re}(s) > 0$
$\cos(bt)$	$s/(s^2+b^2)$	$\mathrm{Re}(s) > 0$
$\sinh(bt)$	$b/(s^2-b^2)$	$\mathrm{Re}(s) > \lvert b\rvert$
$\cosh(bt)$	$s/(s^2-b^2)$	$\mathrm{Re}(s) > \lvert b\rvert$
$\sin^2(bt)$	$2b^2/(s(s^2+4b^2))$	$\mathrm{Re}(s) > 0$
$\cos^2(bt)$	$(s^2+2b^2)/(s(s^2+4b^2))$	$\mathrm{Re}(s) > 0$
$t \cdot \sin(bt)$	$2bs/(s^2+b^2)^2$	$\mathrm{Re}(s) > 0$
$t \cdot \cos(bt)$	$(s^2-b^2)/(s^2+b^2)^2$	$\mathrm{Re}(s) > 0$
$\sin(bt)/t$	$\arctan(b/s)$	$\mathrm{Re}(s) > 0$

Exponential times trigonometric (damped)

f(t)	F(s) = $\mathscr{L}\{f(t)\}$	Valid for
$e^{at} \cdot \sin(bt)$	$b/((s-a)^2+b^2)$	$\mathrm{Re}(s) > a$
$e^{at} \cdot \cos(bt)$	$(s-a)/((s-a)^2+b^2)$	$\mathrm{Re}(s) > a$
$e^{at} \cdot \sinh(bt)$	$b/((s-a)^2-b^2)$	$\mathrm{Re}(s) > a + \lvert b\rvert$
$e^{at} \cdot \cosh(bt)$	$(s-a)/((s-a)^2-b^2)$	$\mathrm{Re}(s) > a + \lvert b\rvert$

Step and impulse

f(t)	F(s) = $\mathscr{L}\{f(t)\}$	Valid for
$u(t-c)\ (c \geq 0)$	e^{-cs}/s	$\mathrm{Re}(s) > 0$
$\delta(t)$	1	all s
$\delta(t-c)\ (c \geq 0)$	e^{-cs}	all s

Properties

Property	Time domain	s-domain
Linearity	$a \cdot f(t) + b \cdot g(t)$	$a \cdot F(s) + b \cdot G(s)$
Frequency shift	$e^{at} \cdot f(t)$	$F(s-a)$
Time shift	$f(t-c) \cdot u(t-c)$	$e^{-cs} \cdot F(s)$
Scaling (c > 0)	$f(ct)$	$(1/c) \cdot F(s/c)$
Multiplication by t^n	$t^n \cdot f(t)$	$(-1)^n \cdot d^n F/ds^n$
First derivative	$f'(t)$	$s \cdot F(s) - f(0)$
Second derivative	$f''(t)$	$s^2 \cdot F(s) - s \cdot f(0) - f'(0)$
nth derivative	$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \ldots - f^{(n-1)}(0)$
Integration	$\int_0^t f(\tau)\, d\tau$	$F(s)/s$
Division by t	$f(t)/t$	$\int F(\sigma)\, d\sigma$, σ from s to ∞
Convolution	$(f * g)(t) = \int_0^t f(\tau) \cdot g(t-\tau)\, d\tau$	$F(s) \cdot G(s)$
Periodic, period T	$f(t+T) = f(t)$	$(1/(1-e^{-sT})) \cdot \int_0^T e^{-st} f(t)\, dt$

Initial and final value theorems

Initial value: $\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} s \cdot F(s)$

Final value: $\lim_{t \to \infty} f(t) = \lim_{s \to 0} s \cdot F(s)$, valid only if every pole of $s \cdot F(s)$ has a negative real part.